1st Junior Penchick Online Mathematical Olympiad

Final Stage, Junior Division

(Ages 16 and below)

4 July 2026

INSTRUCTIONS AND INFORMATION

1. **There are 17 questions to be answered in 120 minutes, split into two parts.** The highest possible score for this contest is 65 points.
   - *Part I (short answer):* Submit the best answer to each question. Three points are given for a correct answer, for a total of 45 points for this section.

     For this paper, all answers are **integers** from 0 to 999 inclusive.
   - *Part II (proof):* The last 2 questions require you to show full solutions in order to get full credit. You can gain a maximum of 10 points for each question depending on the accuracy and completeness of your submission, for a total of 20 points for this section.
2. **Always exhibit academic honesty during the contest.** Do not use calculators, protractors, graph paper, search engines, artificial intelligence, and other external applications, resources, or help while answering the contest. Rulers, compasses, and blank or lined paper are allowed. Cheating in any form is strictly prohibited and will be acted upon based on the violation system.

Enjoy the contest!

Part I: Short-Answer Questions

Submit the best answer to each question. Three points are given for a correct answer, for a total of 45 points for this section.
For this paper, all answers are **integers** from 0 to 999 inclusive.

1. Hinata keeps on improving in volleyball every day! Starting on the third day, he scores a number of points equal to the sum of the points scored in the previous two days. On the first day, he scores no points. If on the fifth day he scored 9 points, find the total number of points Hinata scored over the entire week. The week starts on the first day and ends on the seventh day.

2. Find the sum of all real values of x such that $|x^2 - 13x| + 63 = 11x$.

3. Let $\triangle ABC$ be a triangle with $\angle BAC = 67^\circ$. Then, let D be the foot of the altitude from B to $\overline{AC}$, and E be on $\overline{BC}$ such that $DE = DC$. If $\angle EDB = 36^\circ$, what is the measure of $\angle ABC$ in degrees?

4. Chubby has been captured and blindfolded by Penrick! Penrick tells Chubby that he wrote down three monic quadratics with roots $a+b$, $b+c$ and $c+a$ in some order, where a, b, and c are positive real numbers. Chubby must find the sum of all the constant terms in the three quadratics.

   Chubby's blindfold partially slips off, revealing that the linear terms of each quadratic are $-9x$, $-10x$ and $-11x$. After a bit of thinking, Chubby tells Penrick the correct answer and goes out of the room, frolicking. What number did Chubby say to Penrick?

   *A **monic** quadratic is a quadratic polynomial in the form $x^2 + bx + c$ for real numbers b and c.*

5. Let x be a positive integer. An x by $(x+1)$ by $(2x+1)$ cuboid is made of $x(x+1)(2x+1)$ unit cubes. At least 300 of the unit cubes touch exactly five other small cubes face-to-face. Find the smallest possible volume of the cuboid.

6. The International Penchicken Bank issues gold nuggets of four different weights: 5, 11, 13, and 17 chicks. Determine the greatest total combined weight of gold nuggets that cannot be obtained using any combination of these nuggets.

7. An integer $N < 2026$ is divisible by 67 times the sum of its digits. Find the number of possible values of N.

8. Let $O(0,0)$ be the origin of the Cartesian plane. The line tangent to the circle with the equation $x^2 + y^2 = 40$ at the point $(2,6)$ crosses the x-axis at a point P. Another line tangent to the circle meets the circle at $(-6,-2)$ and intersects the y-axis at Q. The two tangents intersect at R. Find the area of $OPQR$.

9. Pelican has a deck of 16 cards numbered from 1 to 16, and the cards are stacked in ascending order of their number (with 1 on top). He attempts to shuffle the deck as follows:

1. He separates the deck into two smaller decks, one containing the top eight cards and the other containing the bottom eight cards.
2. He then randomly chooses one deck from the two decks, then takes the top card from that deck. He repeats this process, putting the card that he chooses each time on top of the previous card he chose, until he forms a full deck of 16 cards.

He does this process exactly twice. The probability that card 1 is somewhere above card 2 in the resulting deck is equal to $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $m+n$.

10. Penrick has 15 identical types of fish to give to Chubby, Caitlin, Delta, and Yuler. Each penchick must receive at least one fish, and Chubby may receive at most 7 fish. How many ways can the penchicks receive fish?

11. Let a be a real number. Suppose the quadratic equation

$$x^2-(2a+3)x+a^2+3a+1=0$$

has two real roots r and s. It is known that the quadratic equation x^2-rx+s has one root. Find the value of rs.

12. Let a and b be positive integers, and let

$$n=\gcd(a,b)+\operatorname{lcm}(a,b)+a+b.$$

Determine the last 3 digits of the sum of all possible distinct values of n less than 2026.

13. Two robots, Travistron and Phoebot, each independently pick a random positive integer from 1 to 127 (in base ten), inclusive, which they then write in binary (without leading zeros). The probability that the binary representation of Travistron's number has more digits than that of Phoebot's number is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

14. Thirty penchicks, numbered 1 through 30, each write down all the positive divisors of their assigned number. What is the sum of all the numbers they wrote?

15. Let $\triangle ABC$ be a triangle where $AB=16$, $BC=12$, and $AC=20$. Let the perpendicular bisector of $\overline{AC}$ intersect $\overline{AB}$ at D and the circumcircle of $\triangle ABC$ at E and F. Find the value of $|DF-DE|$.

The ***circumcircle*** *of a triangle $\triangle XYZ$ is the circle that passes through the points X, Y, and Z.*

Part II: Proof Questions

The last 2 questions require you to show full solutions in order to get full credit. You can gain a maximum of 10 points for each question depending on the accuracy and completeness of your submission, for a total of 20 points for this section.

1. Let $ABCD$ be a rectangle. Let E be a point on segment $\overline{BC}$ such that there exists a point F such that F lies on segment $\overline{CD}$ such that $\overline{BF} \perp \overline{AE}$. Prove that $DF = CE$ if and only if $ABCD$ is a square.

   Note that "if and only if" means you must prove both directions: you must show that $DF = CE$ if $ABCD$ is a square and that $ABCD$ is a square if $DF = CE$.

2. Delta and Chubby are playing a game with a jar containing 48 marbles. Delta goes first, and they take turns removing 2^n marbles for any integer $n \geq 0$ (such that 2^n is less than the number of marbles currently in the jar). For example, Delta can choose to take 1, 2, 4, 8, 16, or 32 marbles at first. A player wins if their final turn leaves the jar with no marbles; however, a player loses if they can no longer take a valid number of marbles from the jar. Can Chubby guarantee a win?

   

   To get full credit for this question, you must give the correct answer, and do the following:

   - *If your answer is "yes", show a winning strategy that Chubby can use, and prove that it can make him win no matter what moves Delta makes.*
   - *If your answer is "no", give a full proof on why Chubby cannot guarantee a win.*

Answer Key

Part I: Short-Answer Questions

1. 60

2. 30

3. 50

4. 74

5. 840

6. 19

7. 6

8. 300

9. 26

10. 329

11. 2

12. 154

13. 169

14. 762

15. 15