1st Junior Penchick Online Mathematical Olympiad

Final Stage, Primary Division

(Ages 12 and below)

4 July 2026

INSTRUCTIONS AND INFORMATION

1. **There are 17 questions to be answered in 120 minutes, split into two parts.** The highest possible score for this contest is 65 points.

   - *Part I (short answer):* Submit the best answer to each question. Three points are given for a correct answer, for a total of 45 points for this section.

     For this paper, answers are either **whole numbers** or **fractions** (in the form of common fractions, mixed fractions, or decimals). All answers must be exact and in their simplest form; otherwise, they will be marked as incorrect. For example, if the answer is $\frac{5}{3}$, then the answers $\frac{5}{3}$, $1\frac{2}{3}$, and $1.\overline{6}$ are accepted (for the decimal, the answer is accepted as long as it is indicated that the decimal is repeating, for example $1.66666\ldots$). However, the answer $\frac{10}{6}$ is not accepted because it is not simplified, and the answer 1.67 is not accepted because it is an estimation of the answer.

   - *Part II (proof):* The last 2 questions require you to show full solutions in order to get full credit. You can gain a maximum of 10 points for each question depending on the accuracy and completeness of your submission, for a total of 20 points for this section.

2. **Always exhibit academic honesty during the contest.** Do not use calculators, protractors, graph paper, search engines, artificial intelligence, and other external applications, resources, or help while answering the contest. Rulers, compasses, and blank or lined paper are allowed. Cheating in any form is strictly prohibited and will be acted upon based on the violation system.

Enjoy the contest!

Part I: Short-Answer Questions

Submit the best answer to each question. Three points are given for a correct answer, for a total of 45 points for this section.
All answers must be exact and in their simplest form; see the front page of this questionnaire for more details.

1. Calculate $22\frac{69}{7}+42\frac{68}{7}+67\frac{67}{7}+69\frac{6}{7}$.

2. Seven real numbers are written in ascending order. The average of the first six numbers is 7, and the average of the last six numbers is 9. Find the difference between the largest and the smallest of the seven numbers.

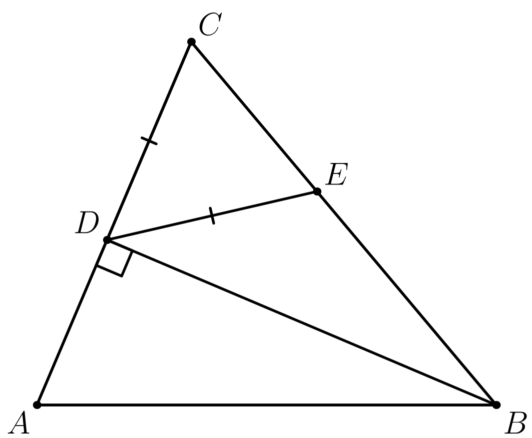
3. Four queens act in turn on a pile of tarts: Diamonds, Hearts, Spades, then Clubs. On her turn, each queen takes 1 tart, then takes three quarters of what remains. After the Queen of Clubs acts for the first time, no tarts are left. If every queen takes a whole number of tarts, how many tarts were there initially?

4. Let $G=\gcd(2026,2027)$ and $L=\operatorname{lcm}(2026,2027)$. Find the last three digits of

$$\gcd(L,G)+\operatorname{lcm}(L,G).$$

*Here, $\gcd(a,b)$ and $\operatorname{lcm}(a,b)$ refer to the **greatest common divisor** and **least common multiple** of a and b, respectively.*

5. Let $\triangle ABC$ be a triangle with $\angle BAC=67^\circ$. Then, let D be the foot of the altitude from B to $\overline{AC}$, and E be on $\overline{BC}$ such that $DE=DC$. If $\angle EDB=36^\circ$, what is the measure of $\angle ABC$ in degrees?



*The **foot** D of an **altitude** $\overline{BD}$ is the point D on $\overline{AC}$ such that $\overline{BD}$ is perpendicular to $\overline{AC}$.*

6. Renren drops several pairs of infinitely long chopsticks. Each chopstick is parallel to exactly one other chopstick, and at every intersection, exactly two chopsticks meet. If there are 180 intersections in total, how many pairs of chopsticks were dropped?

7. Anoupel builds a solid with some identical tiles, shaped like an equilateral triangle. If she looks the solid from every vertex where only the faces touching that vertex can be seen, she sees a quadrilateral. How many tiles did she use?

8. Ten penchicks compete in a race, each from exactly one region. There are no ties. The table below shows each penchick's overall rank and the rank within their respective region. Given that each region is **indistinguishable** from another, how many possible ways could the penchicks be grouped into regions consistent with the table's ranks?

Overall Rank	Name	Regional Rank
1	Blizzy	1
2	Chubby	1
3	Decoy	2
4	Delta	1
5	Epsilon	2
6	Pelican	3
7	Penelope	1
8	Renren	1
9	Superpenrick	2
10	Yuler	3

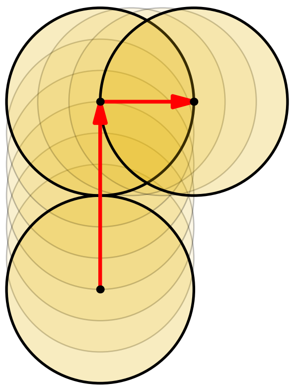
One possible way to group the penchicks is {Blizzy, Epsilon, Pelican}, {Chubby, Decoy}, {Delta}, {Penelope}, {Renren, Superpenrick, Yuler}.

9. At Penchick's Burger Shack, burgers, fries and drinks are sold. Two different families of penchicks order the following sets of food:

- 3 burgers, 4 drinks, and 5 fries which cost 2,400 pesos in total; and
- 2 burgers, 3 drinks, and 4 fries which cost 1,700 pesos in total.

How much will 1 drink and 2 fries cost?

10. In the game *OneShot*, Niko carries a lightbulb that illuminates a circle of radius 6. If Niko moves 12 units north, then 6 units east, the total illuminated area is $a+b\pi$, where a and b are positive integers. Find $a-b$.

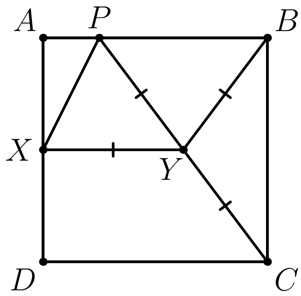


11. Welcome to your daily dose of penchicks. Today, we are covering the annual penchick race. Penrick, Renren, and Blizzy and 6 others are racing each other over a meter race. All of them run at a constant speed throughout the race. Penrick beats Blizzy by 100 meters. Blizzy beats Renren by 200 meters. By how many meters does Penrick beat Renren?

12. A natural number has 12 factors and ends in 00. What is the smallest number of factors that its cube could have?

13. The International Penchicken Bank issues gold nuggets of four different weights: 5, 11, 13, and 17 chicks. Determine the greatest total combined weight of gold nuggets that cannot be obtained using any combination of these nuggets.

14. In the figure below, $ABCD$ is a square with side length 8. Let X be the midpoint of $\overline{AD}$, and Y be the point inside $ABCD$ such that $BY = CY = XY$. Lastly, let P be the point on $\overline{AB}$ where $PY = XY$. Find XP^2.



15. Renren owns two objects made from the same underlying material, and both start from the same initial value. Each object loses value over time at a constant rate proportional to its volume (the volume stays constant). One object is cube-shaped with volume $52\,\text{cm}^3$. After 2.5 minutes, its value is 57 units. After 8 minutes, its value is 35 units. The second object is spherical. It loses all of its units after a minute. What is the volume of the spherical object (in cm^3)?

Part II: Proof Questions

The last 2 questions require you to show full solutions in order to get full credit. You can gain a maximum of 10 points for each question depending on the accuracy and completeness of your submission, for a total of 20 points for this section.

1. A positive integer is **Penrick-approved** if both of the following conditions are satisfied:

- All of its digits are nonzero.
- The integer is not divisible by any of its digits.

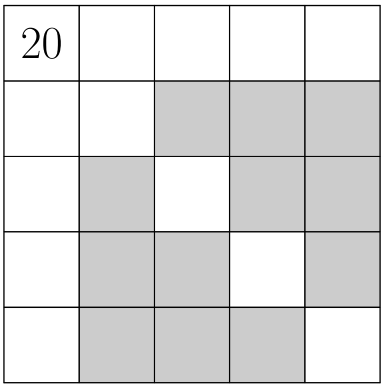
For example, 67 is Penrick-approved, since it is neither divisible by 6 nor by 7.

(a) Show that a Penrick-approved number must not contain 1.

(b) Show that for all integers $n \geq 2$, the number $\underbrace{55\ldots5}_{n-1 \text{ digits}}4$ is Penrick-approved.

(c) Show that for all integers $n \geq 2$, there exist at least three *more* Penrick-approved numbers with exactly n digits, other than $\underbrace{55\ldots5}_{n-1 \text{ digits}}4$.

2. The numbers $1, 2, \ldots, 25$ are placed in a 5×5 grid, each number appearing in exactly one cell. The sums of the numbers in every row, every column, and both of the main diagonals are equal. If 20 is assigned to the upper-left cell, determine, with proof, all possible sums of the shaded cells below.



To get full credit for this question, you must give the correct answer, and do the following:

- *Show that all possible sums you provided are possible to achieve.*
- *Explain why there are no other possible sums.*

Answer Key

Part I: Short-Answer Questions

1. 230

2. 12

3. 85

4. 703

5. 50

6. 10

7. 8

8. 48

9. 300

10. 135

11. 298

12. 70

13. 19

14. 20

15. 871