1st Junior Penchick Online Mathematical Olympiad

Qualifying Stage, Sample Problems

2026

INSTRUCTIONS AND INFORMATION

Each paper has **seven sample problems** that you can use to prepare for the JPOMO. The answers and solutions to each item can be found starting on page 4.

The sample problems (and the problems in the actual test) are of two types:

- *Multiple-choice:* Each question has four choices (A, B, C, and D), and only one of these choices is correct. Submit the best answer to each question.
- *Short-answer:* Submit the best answer to each question.

  For papers A and B, answers are either **whole numbers** or **fractions** (in the form of common fractions, mixed fractions, or decimals). All answers must be exact and in their simplest form; otherwise, they will be marked as incorrect. For example, if the answer is $\frac{5}{3}$, then the answers $\frac{5}{3}$, $1\frac{2}{3}$, and $1.\overline{6}$ are accepted (for the decimal, the answer is accepted as long as it is indicated that the decimal is repeating, for example $1.66666\ldots$). However, the answer $\frac{10}{6}$ is not accepted because it is not simplified, and the answer 1.67 is not accepted because it is an estimation of the answer.

  For papers C, D, and E, all answers are **integers** from 0 to 999 inclusive.

Enjoy the problems!

Contents

1 Paper A

1. If ★ $=9$ and ■ $=$ ★ -4, then find the value of

$$★ + 4■ + 7.$$

(a) 28 (b) 31 (c) 36 (d) 41

2. April Fool's Day 2026, which falls on April 1, 2026, is a Wednesday. On what day of the week will Talairon's Fiesta, held on March 3, 2029, fall?

(a) Friday (b) Saturday (c) Sunday (d) Monday

3. Blizzy picked some roses to give to Pelican, Chubby, and Renren. Blizzy first gave $\frac{1}{4}$ of the roses to Pelican. From the remaining roses, Blizzy then gave $\frac{3}{5}$ to Chubby. Finally, all the remaining roses were given to Renren. Which of the following could be the total number of roses Blizzy picked at the start?

(a) 25 (b) 30 (c) 36 (d) 40

4. A secret 5-digit number is written on the board. Blizzy gives the following clues:

1. The sum of all digits is 25.
2. The first digit is 1 more than the last digit.
3. The middle three digits form a 3-digit number divisible by 5.
4. The second digit is twice the fourth digit.

What is the secret number?

(a) 90808 (b) 82717 (c) 80854 (d) 34567

5. Donuts are sold in packs of 5 or 9. Chubby multiplies the number of donuts in each pack and gets 2025. How many donuts did he buy?

6. Penrick and Renren are sharing a large chocolate bar. First, Penrick eats $\frac{3}{7}$ of the bar. Then, Renren eats half of what is left. Let n be the fraction of the chocolate bar that they have eaten altogether. What is $100 \times n$ to the nearest integer?

7. Light the Persian spends most of the day sleeping so that it has enough energy to play. In one day, Light eats:

- 75 kcal for breakfast
- 85 kcal for lunch
- 90 kcal for dinner

Light's body absorbs 90% of the calories (kcal) eaten. All the absorbed energy is used for playing. Light spends 25 kcal for every hour of play. When Light is not playing, it sleeps and uses no energy. If Light uses exactly all the absorbed calories (kcal) in one day, what is the ratio of the number of hours Light sleeps to the number of hours Light plays?

2 Paper B

1. A penchick can make one of three sounds: squawk, chirp, or quack. It makes a sequence of four sounds, but it cannot make the same sound twice in a row. How many different sequences of sounds are possible?

(a) 12 (b) 24 (c) 64 (d) 81

2. The following numbers are to be ordered from smallest to largest:

$$X = 3^{4^4}, \quad Y = 4^{3^4}, \quad Z = 4^{4^3}$$

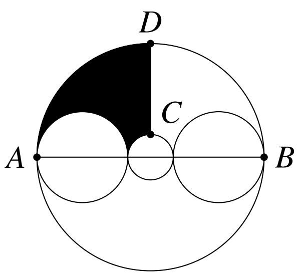
Which of the following gives the correct order?

(a) $X < Y < Z$ (b) $X < Z < Y$ (c) $Z < Y < X$ (d) $Y < Z < X$

3. The six-digit number $\overline{36A74A}$ is divisible by 36, where A is a digit. What is A?

(a) 2 (b) 4 (c) 6 (d) 8

4. Shown in the diagram is a circle with diameter $AB = 10$. Inside it are three circles whose diameters lie on AB, with the two circles on the side having radius 2 and the circle on the center having radius 1. If $\overline{CD}$ divides the region inside the large circle and above the smaller circles into two equal parts (one of which is shaded), find the area of the shaded region.



(a) $\frac{5}{4}\pi$ (b) 4π (c) $\frac{25}{4}\pi$ (d) 8π

5. Yuler writes a long number that starts with the digit 4 such that every pair of consecutive digits always forms a prime number. If no digit appears more than twice, what is the largest number he can write?

6. How many integers less than or equal to 2026 contain both a 6 and 7 with at least one 6 preceding at least one 7 (not necessarily adjacent to each other)?

7. There are four subjects that each person is assigned to: Algebra, Geometry, Combinatorics, and Number Theory. Each person gets assigned exactly 1 subject. If the ratio of people assigned Algebra to Geometry is 3 : 5, the ratio of people assigned Number Theory to Algebra is 4 : 7 and the ratio of people assigned Geometry to the total amount of people is 1 : 4, what is the ratio of people assigned Combinatorics to Number Theory?

3 Paper C

1. Which of these is a right-angled triangle?

Option	Side 1	Side 2	Side 3
A	0	5	5
B	3	5	6
C	5	12	13
D	17	21	30

(a) A (b) B (c) C (d) D

2. What whole number, when raised to the 6th power, equals 4^9?

(a) 2 (b) 8 (c) 12 (d) 16

3. Let x be a real number satisfying

$$\frac{x^2-5x+6}{x-2}+\frac{x^2-3x-4}{x+1}=2x-1.$$

Find the sum of the squares of all real solutions.

(a) 0 (b) 1 (c) 4 (d) 5

4. A function $a(i,j)$ satisfies the following rules:

$$a(i,j)=a(i-1,j)+a(i,j-1),$$

and for all nonnegative $i,j\neq 1$,

$$a(0,j)=0 \quad \text{and} \quad a(i,0)=0.$$

If $a(1,1)=1$, then find the remainder when $a(20,26)$ is divided by 3.

(a) 0 (b) 1 (c) 2 (d) 3

5. A polygon is not convex. Its interior angles are

$$62^\circ,\ 47^\circ,\ 169^\circ,\ 200^\circ,\ 200^\circ,\ 31^\circ,\ 153^\circ,\ 138^\circ,\ 91^\circ,\ x^\circ.$$

Find the last interior angle x°, where x is a positive integer less than 360.

6. A box contains 5 red balls, 4 blue balls, and 3 green balls. Five balls are drawn from the box at random, one after another, without replacement. What is the probability that the drawn set contains at least one ball of each color? If the probability is $\frac{m}{n}$ in lowest terms, find $m+n$.

7. A line with slope 3 passes through the point $(x,\ a+5)$. A second line with its slope and y-intercept being thrice of the first line goes through the point $(x,\ a+25)$. If a third line exists with y-intercept 100 and slope 1 such that it passes through the point $(x,\ a)$, then what is the y-intercept of the first line?

4 Paper D

1. The quadratics x^2+7x+k and x^2+kx+7 have exactly one common root. If $k \neq 7$, what is the value of k?

(a) -8 (b) -6 (c) 6 (d) 8

2. Five penchicks, including Pelican and Chubby, are seated in a row of five chairs. In how many ways can they be arranged so that Pelican and Chubby are not seated next to each other?

(a) 48 (b) 72 (c) 96 (d) 120

3. The number 30! can be simplified as which of these options:

(a) $2^{26} \cdot 3^{14} \cdot 5^7 \cdot 7^4 \cdot 11^2 \cdot 13^2 \cdot 17 \cdot 19 \cdot 23 \cdot 29$

(b) $9! \cdot 21!$

(c) $2^{26} \cdot 3^{15} \cdot 5^6 \cdot 7^4 \cdot 11^2 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29$

(d) $11! \cdot 19!$

4. Let θ be an angle on the unit circle such that

$$\sin\theta + \cos\theta = \frac{1}{3}.$$

Find the value of $\sin(2\theta)$.

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{2}{3}$ (c) $-\frac{4}{9}$ (d) $-\frac{8}{9}$

5. A safe uses a code made from the value of the expression

$$2^{67}+3^{67}+5^{67}$$

If the remainder of the expression when divided by 8 and 9 are x and y, respectively, then find the value of $6x+7y$.

6. Let $P(x) = ax^4+x^3+bx^2+cx+168$ where a, b, and c are real numbers. If three of the roots of $P(x)$ are -2, 2, and 6, what is the absolute value of the other root of $P(x)$?

7. In triangle ABC, point P lies inside the triangle. Lines AP, BP, and CP meet sides BC, CA, and AB at D, E, and F, respectively. Suppose the areas of triangles ABP, BCP, and CAP are 18, 24, and 30, respectively. If the ratio $BF : FA$ can be expressed as $\frac{a}{b}$ for some relatively prime integers a and b, what is the value of $a+b$?

5 Paper E

1. A quadrilateral has sides 5, 6, 7, and x, such that the side x is opposite the side of length 5. If x is an integer, how many values can x take?

(a) 11 (b) 13 (c) 15 (d) 17

2. Given that $1434_x = 100001_3$, find x.

(a) 4 (b) 5 (c) 6 (d) 7

3. A conic section has a focus at $F(4,-2)$ and a directrix given by the line

$$2x - y + 6 = 0.$$

The conic passes through the point $P(1,1)$. Which of the following is the equation of the conic section?

(a) $(x-4)^2 + (y+2)^2 = 25$

(b) $(x-1)^2 + (y-1)^2 = 10$

(c) $(x-4)^2 + (y+2)^2 = \frac{25}{4}$

(d) $(x-4)^2 + (y+2)^2 = \frac{(2x-y+6)^2}{5}$

4. Let

$$f(x) = \sin(2026x) \quad \text{and} \quad g(x) = \frac{x}{2026}.$$

How many times do the graphs of $y = f(x)$ and $y = g(x)$ intersect for

$$0 \le x \le 2\pi\,?$$

(a) 4051 (b) 4052 (c) $2026^2 - 1$ (d) 2026^2

5. If positive integers x, y exist such that

$$\log(4x) + 2\log(\log y) = 5,$$

then find the sum of the least and greatest possible values of $\log y$.

6. Find the largest prime divisor of the number $2^{24} + 3^{12} - 2 \cdot 3^6 \cdot 2^{12}$.

7. A penchick has 10 fish in front of him. He can either swallow a fish whole or peck at it. However, he cannot swallow two fish in a row. In how many different ways can he eat all 10 fish?