



2nd Penchick Open Mathematical Olympiad

Qualifying Stage, 9-11 October 2026

PART I. Choose the best answer. Each correct answer is worth two points.

1. Pelican has a playlist containing 12 songs, each lasting for exactly 2, 3, 4, or 5 minutes. There are an equal number of 2-minute and 4-minute songs, and there are twice as many 3-minute songs as 5-minute songs. If the total runtime of the playlist is exactly 40 minutes, how many 4-minute songs are in the playlist?

(a) 2 (b) 3 (c) 4 (d) 5

2. Suppose Max has a flock of n peculiar penchicks for some positive integer n , and he chooses two of those penchicks to take on a road trip. What is the minimum possible value of n such that Max has at least 1000 ways of choosing the two penchicks?

(a) 44 (b) 45 (c) 46 (d) 47

3. Four cards are labeled with the numbers 2, 3, 5, and 7. Matt places the four cards in the four boxes, in some order, to satisfy the following inequality:

$$\frac{\square}{\square} < \frac{\square}{\square}.$$

If each card is used exactly once, how many ways is this possible?

(a) 6 (b) 8 (c) 10 (d) 12

4. Let $f(x) = \frac{2x+3}{x-1}$ and $g(x) = 2f(3x-2) - 5$. If $g(x)$ can be written as $a + \frac{b}{x-c}$, where a , b , and c are constants and $b \neq 0$, what is the ordered pair (c, a) ?

(a) $(1, -1)$ (b) $(1, -3)$ (c) $\left(\frac{1}{3}, -1\right)$ (d) $\left(\frac{1}{3}, -3\right)$

5. Yuler needs to study for an upcoming math contest! On the n th day, he correctly computes the remainder m of $n^2 + n$ when divided by 7, then solves m problems on that day. How many problems does he solve altogether for the first 28 days?

(a) 76 (b) 80 (c) 84 (d) 88

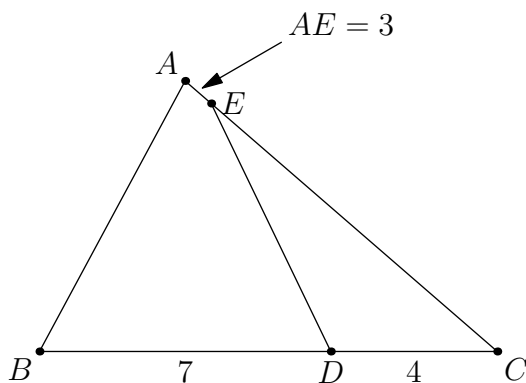
6. Let k be a four-digit positive integer such that $2025 - k$, $2015 - k$, and $2005 - k$ are all prime numbers. Find the sum of the digits of k .
- (a) 4 (b) 6 (c) 8 (d) 10
7. How many ways can Blizzy, Chubby, Delta, Pelican, Renren, and Superpenrick sit around a circular table if Blizzy, Chubby, Delta, and Pelican must not appear in that order when read clockwise or counterclockwise? The restriction applies even when these four are not seated consecutively. Arrangements that differ only by rotation are considered the same; reflected arrangements are considered different.
- (a) 80 (b) 90 (c) 100 (d) 110
8. A multiplication table of the numbers 1 to 50 is shown. Let S be the sum of all odd entries in the body of the complete table, counting each cell separately. What is the remainder when S is divided by 1000?

$\times$	1	2	3	$\dots$	48	49	50
1	1	2	3	$\dots$	48	49	50
2	2	4	6	$\dots$	96	98	100
3	3	6	9	$\dots$	144	147	150
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
48	48	96	144	$\dots$	2304	2352	2400
49	49	98	147	$\dots$	2352	2401	2450
50	50	100	150	$\dots$	2400	2450	2500

- (a) 125 (b) 375 (c) 625 (d) 875
9. Let $ABCDEFGHIJKL$ and $ABRQPONM$ be a regular dodecagon and a regular octagon, respectively, with their vertices listed in cyclic order. The polygons share side $\overline{AB}$ and lie on opposite sides of line AB . Let ABS be an equilateral triangle with S inside the octagon. Which of the following angles is not equal to 30° ?
- (a) $\angle SLB$ (b) $\angle BED$ (c) $\angle OHC$ (d) $\angle SMC$
10. Find the sum of all positive rational numbers less than 15 whose denominator is 45 when written in simplest form.
- (a) 1800 (b) 2100 (c) 2400 (d) 2700

PART II. Choose the best answer. Each correct answer is worth three points.

1. Let ABC be a triangle, and let D and E be points on segments $\overline{BC}$ and $\overline{AC}$, respectively, such that $BD = 7$, $DC = 4$, $AE = 3$, and the area of quadrilateral $ABDE$ is twice the area of $\triangle DEC$. Find the length of $\overline{CE}$.



- (a) 24 (b) 27 (c) 30 (d) 33
2. If the 5-digit number $\overline{YULER}$, where the five letters represent distinct digits and $Y \neq 0$, is divisible by 990, what is the greatest possible value of $Y + U + L + E + R$?
- (a) 18 (b) 21 (c) 24 (d) 27
3. In right triangle ABC , we have $AB = 80$, $BC = 18$, and $AC = 82$. Let M be the midpoint of $\overline{AC}$. Let D be a point on minor arc $\widehat{BC}$ of the circumcircle of $\triangle ABC$ such that line MD is perpendicular to line BC . If line MD intersects line BC at E and the circumcircle again at a point $F \neq D$, what is EF ?
- (a) 40 (b) 41 (c) 80 (d) 81
4. Let $ABCDEF$ be a regular hexagon. Let G lie on segment $\overline{AF}$ and H on segment $\overline{EF}$ such that $AG = EH$. If the area of $\triangle FGH$ is 96 square units and the area of $\triangle GHC$ is 384 square units, find the area of $\triangle GAB$.
- (a) 30 (b) 36 (c) 60 (d) 72
5. Suppose $P(x)$ is a monic polynomial (a polynomial with leading coefficient 1) that satisfies the following conditions:
- $P(0) = -16$, $P(1) = -14$, and $P(2) = -10$,
 - $P(x) - x^2 - x + 16$ is divisible by $1 + x + x^2 + \cdots + x^n$ for some positive integer n , and
 - the degree of $P(x)$ is equal to $n + 3$.

For what positive integer n is $P(-2) = 2026$?

- (a) 4 (b) 5 (c) 6 (d) 7

6. A convex polyhedron (three-dimensional shape) has 15 vertices and 14 faces. Each face is a regular polygon: a square, triangle, or hexagon. Exactly one face is a hexagon. How many square faces does the polyhedron have?
- (a) 6 (b) 7 (c) 8 (d) 9
7. A positive integer is called *wonderful* if the product of its digits is between 20 and 26, inclusive. For example, 1226 is wonderful because $1 \cdot 2 \cdot 2 \cdot 6 = 24$, which is between 20 and 26, inclusive, and 242 is not because $2 \cdot 4 \cdot 2 = 16$. How many wonderful positive integers are there that are less than 2026?
- (a) 87 (b) 104 (c) 123 (d) 150
8. Chubby colors each of the eight labeled vertices of a regular octagon *PENRICKS* red, blue, or green, with vertex *R* required to be red. A coloring is called *penricky* if no two adjacent vertices are red. How many penricky colorings are there? Colorings are different if at least one labeled vertex has a different color.
- (a) 328 (b) 448 (c) 496 (d) 656
9. Let N be the number of ways to arrange the numbers 1 to 15 in a row, using each number exactly once, so that the sum of every three consecutive entries is a multiple of 3. What is the remainder when N is divided by 13?
- (a) 3 (b) 6 (c) 7 (d) 11
10. Let $P(x)$ be a sixth-degree polynomial with real coefficients such that $P(3) = 3$, $P(5) = 5$, $P(10) = 10$, $P(1) = -1$, and $P(x) \leq x$ for every real number x . What is the constant term of $P(x)$?
- (a) $-\frac{625}{36}$ (b) $-\frac{125}{72}$ (c) $-\frac{125}{36}$ (d) $-\frac{625}{72}$

PART III. The answer to each item is a positive integer from 1 to 999. Each correct answer is worth five points.

1. Let P be the perimeter of an equilateral triangle with area 24, and let n be the smallest positive integer such that P^n is an integer. Find the number of positive factors of P^n .
2. Suppose a , b , and c are real numbers such that $a < c$ and

$$\begin{aligned}a + b + c^2 &= 4, \\a + b^2 + c &= 2, \\a^2 + b + c &= 4.\end{aligned}$$

The sum of all possible values of c is $\frac{m + \sqrt{n}}{p}$, where m , n , and p are positive integers with $\gcd(m, p) = 1$. Find $m + n + p$.

3. An airport corridor consists of two equal-length sections. One section has an ordinary floor, while the other has a moving walkway traveling in the direction of travel at 4 km/h. A traveler traverses the entire corridor once, always moving forward and either walking or running. Relative to the surface underfoot, the traveler walks at 5 km/h and runs at 8 km/h. The traveler may switch between walking and running at any point, with no time lost in switching.

Over the entire trip, the traveler spends exactly half of the total travel time walking and half running. Let $v_{\min}$ and $v_{\max}$ be the minimum and maximum possible average travel speeds over the entire corridor, including the boost from the moving walkway. If

$$v_{\max} - v_{\min} = \frac{a}{b} \text{ km/h}$$

where a and b are relatively prime positive integers, find $a + b$.

4. Let ABC be a triangle such that $BC = 14$, $CA = 13$, $AB = 15$. The incircle of $\triangle ABC$ touches BC at D . Let $X \neq A$ be the second intersection of line AD with the circumcircle of $\triangle ABC$, and let Y be the midpoint of the arc BC not containing A . If

$$XY^2 = \frac{a}{b}$$

where a and b are relatively prime positive integers, find $a + b$.

5. Superpenchick is standing at a point P in the interior of a horizontal circular tabletop. His eyes are at a point E directly above P . Looking due north, east, south, and west from P , let N , X , S , and W , respectively, be the points where those four horizontal rays meet the edge of the table. The straight-line distances from Superpenchick's eyes to these points are

$$EN = 19, \quad EX = 23, \quad ES = 29, \quad EW = 23.$$

Superpenchick now chooses a horizontal direction from P uniformly at random from all directions. The ray from P in that direction meets the edge of the table at Q . Let $\theta = \angle PEQ$. If the expected value of $\tan^2 \theta$ is $\frac{a}{b}$ for relatively prime positive integers a and b , find $a + b$.

6. Six penchicks are independently colored pink, grey, or black, with each color equally likely. Initially, each penchick can see the colors of the other five penchicks, but not its own color. Before day 1, Penrick publicly and truthfully announces exactly which of the three colors appear among the six penchicks, but not how many of each color there are.

The penchicks may not communicate. At the end of each day, all penchicks that can determine their own color from their initial observations, the announcement, and the departures on earlier days leave simultaneously. Everyone still in the room sees who leaves. Departures at the end of a day may be used only in making decisions for subsequent days. All penchicks reason perfectly and remember their observations. This and all the rules are common knowledge: everyone knows them, everyone knows that everyone knows them, and so on.

Choose one of the original six penchicks uniformly at random, independently of the coloring. Let T be the number of days it stays in the room, so a penchick that leaves at the end of the first day has $T = 1$. If the expected value of T is $\frac{a}{b}$, where a and b are relatively prime positive integers, find $a + b$.

7. Let ABC be a triangle such that $BC > CA > AB$. Let r , r_A , and r_B be its inradius, A -exradius, and B -exradius, respectively. Suppose that $\angle ABC = 2\angle ACB$, $r_A + r_B = 9r$, and $AC - AB = 6$. Find $BC + CA + AB$.
8. Let P be a monic polynomial (a polynomial with leading coefficient 1) of degree 36 with real coefficients such that:

- all its roots are real,
- the sum of the squares of its 36 roots, counted with multiplicity, is 87,
- $P(1 - x) = P(x)$ and $x^{36}P\left(\frac{1}{x}\right) = P(x)$ for all nonzero real numbers x , and
- $P(2) = 3^{11}$.

Let R and r be the largest and smallest roots, respectively, of P . The minimum possible value of $R - r$ is $a + b \cos(c^\circ)$ for integers a , b , and c with $0 \leq c \leq 90$. Find $100a + 10b + c$.

1 Answers

Part I. (2 points each)

- | | | |
|------|------|-------|
| 1. b | 5. c | 9. d |
| 2. c | 6. a | 10. d |
| 3. d | 7. a | |
| 4. a | 8. c | |

Part II. (3 points each)

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|------|------|-------|
| 1. d | 5. d | 9. b |
| 2. a | 6. d | 10. d |
| 3. d | 7. a | |
| 4. a | 8. d | |

Part III. (5 points each)

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|--------|--------|--------|
| 1. 66 | 4. 42 | 7. 40 |
| 2. 24 | 5. 113 | |
| 3. 109 | 6. 796 | 8. 160 |